MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 1, 2015/2016

EME4076 – MECHANICAL VIBRATIONS

(All sections)

15 OCTOBER 2015
02:30 p.m. – 04:30 p.m.
(2 Hours)

INSTRUCTIONS TO STUDENTS

1. This Question paper consists of 9 pages including cover page with 5 Questions and 3 appendices.

2. Attempt **FOUR** out of **FIVE** questions. All questions carry equal marks and the distribution of the marks for each question is given.

3. Please write all your answers in the Answer Booklet provided.

Question 1

Figure Q1 shows a tail rotor mounted at the end of a long tail boom of a helicopter. The rotor consists of four blades, each of mass 2.1 kg, and an engine box of mass 25 kg. The center of gravity of each blade is 170 mm from the rotational axis. The boom can be modeled as a massless elastic structure and is connected to the fuselage of the helicopter. The natural frequency and damping ratio of the system has been experimentally determined as 150 rad/s and 0.05, respectively. During flight the rotor operates at 900 rpm. Let's consider the following three cases.

(a) If during flight a 75-g particle becomes stuck to blade A, 25 cm from the axis of rotation, determine the steady-state response of the system caused by the resulting rotating unbalance.

[8 marks]

(b) Determine the steady-state response of the system if blade B snaps off during flight.

[8 marks]

(c) Find the steady-state response of the system if 75-g particle sticks to blade A while blade B snaps off simultaneously during flight.

[9 marks]

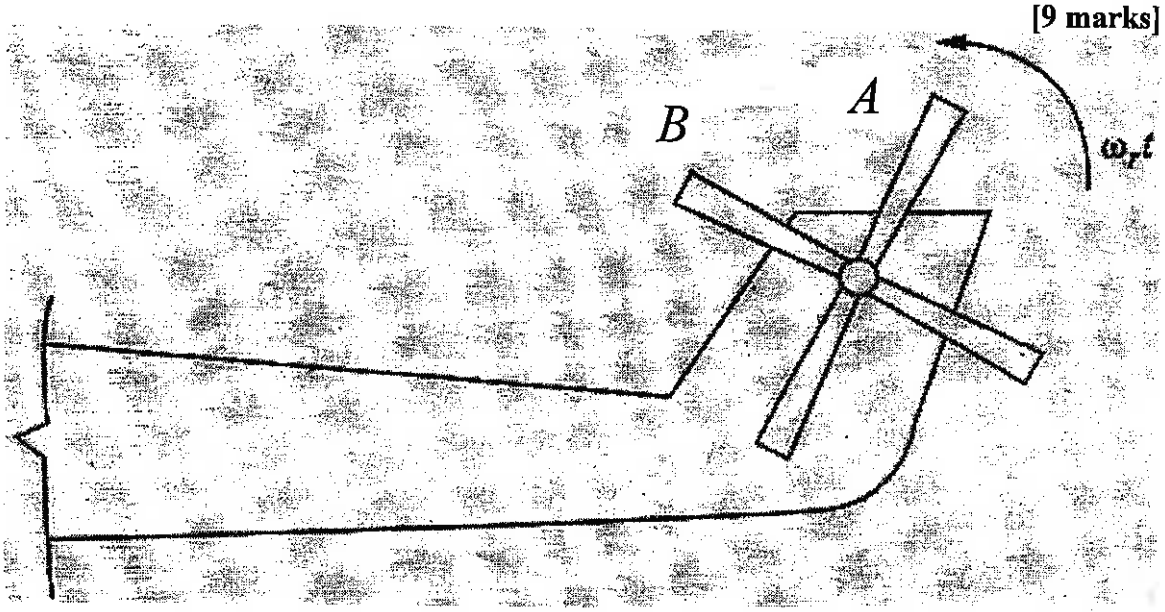


Figure Q1

Continued .....

Question 2

(a) Figure Q2(a) shows the schematic model of a personal computer disk drive system. The disk drive motor is mounted to the computer chassis through an undamped isolation pad. The motor has a mass of 3 kg and operates at 5000 rpm. An unbalance force with magnitude of 1 N is generated due to the slightly eccentricity in the motor. Calculate the value of the stiffness of the isolator needed to provide a 95% reduction in force transmitted to the chassis. And also determine the clearance that is needed in between the motor and the chassis so that the isolator is able to perform its function. As a rule, it should be more than twice the total settling of the isolator.

[12 marks]

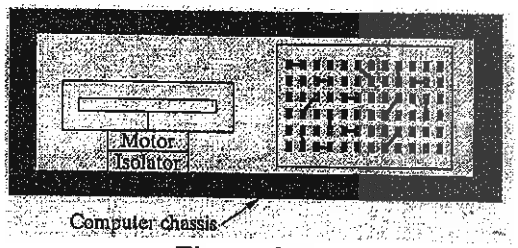


Figure Q2(a)

(b) A printed circuit board (PCB) may be sensitive to vibration because vibration can loosen the soldered joints between the components. Figure Q2(b) shows a 0.45-kg PCB supported by a chassis, which is to be isolated from a nearby unbalance motor by using an isolator. The isolator is made of standard rubber material which has damping ratio of 0.1. The board is 1.6 mm thick, 178 mm wide, 200 mm long, and has a young modulus E of 1.38×10^{10} N/m^2. Ignore the mass of the chassis. Assume the board has negligible damping and can be modeled as a fixed-fixed beam with its mass lumped at its center. If the frequency of vibration created by the motor is 60 Hz, obtain the required values for the isolator stiffness k_2 and damping c such that no more than 20% of this vibration should be allowed to transmit to the board.

[13 marks]

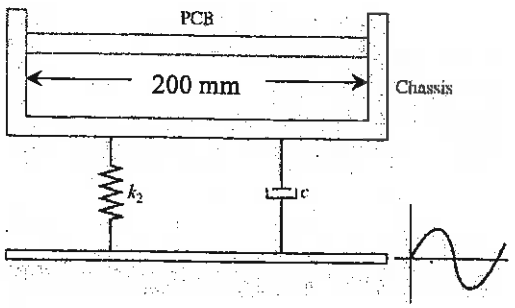


Figure Q2(b)

Continued .....

Question 3

A small trailer, as shown in Figure Q3, and its load have a total mass of 500 kg in which the mass of the wheels has been neglected. During loading, it was found out that each 75 kg added to the load will cause the trailer's springs to sag 3 mm. The trailer is pulled over a corduroy road at a constant speed v. The contour of the road surface can be approximated by a sine curve with an amplitude of 25 mm and a wavelength of 10 m. The radio antenna of the trailer consists of a 1-m long aluminium cantilever beam with a 10 mm×10 mm square cross section. The Young's modulus E and mass density ρ of the aluminium are 70 GPa and 2700 kg/m^3, respectively. Assume that the mass m of the antenna can be lumped at its free end, the damping of the suspension system and antenna of the trailer can be ignored, and the trailer is only vibrated in vertical direction.

(a) If the vertical distance from the roof of the trailer to the free end of the antenna at equilibrium is 10 cm, determine the speed at which the trailer needs to travel such that the free end of the antenna will hit the roof.

[15 marks]

(b) Determine the critical speed of the trailer at which the resonance will occur.

[5 marks]

(c) Determine the speed of the trailer at which the amplitude of vibration of the antenna will be maximum.

[5 marks]

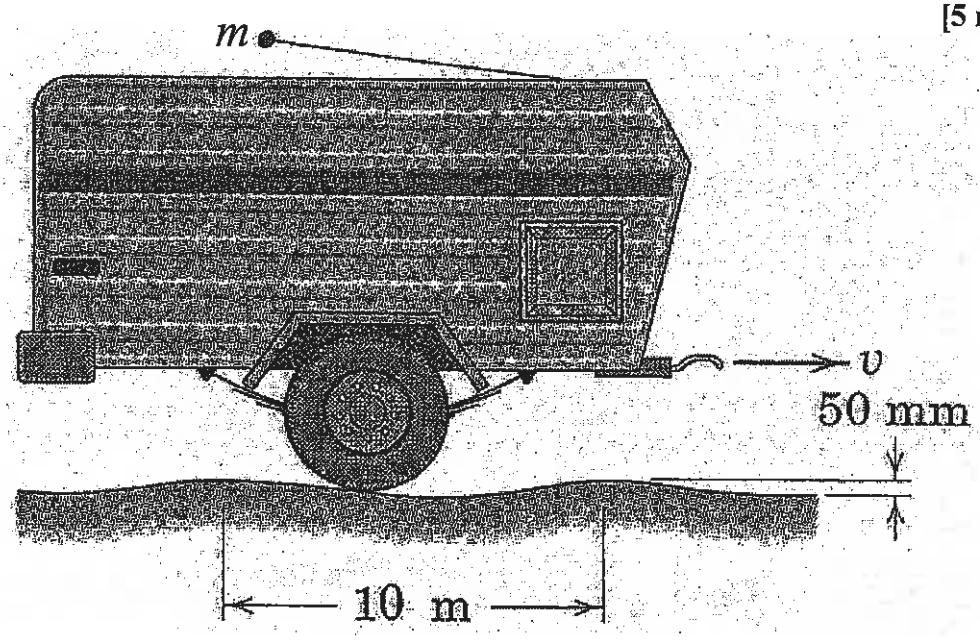


Figure Q3

Continued .....

Question 4

Figure Q4(i) shows a package with mass m which is handled by a worker in a package handling center is accidentally dropped onto a conveyor belt. The conveyor belt can be modelled as a simple supported beam, as shown in Figure Q4(ii), with length l. Knowing that the belt has a static deflection of $\Delta_s = 0.25$ cm when the package is placed at its center. To simulate condition of Figure Q4(i), the package is then dropped from the height $h = 10\Delta_s$ onto the middle of the belt. However, the package remains in contact with the belt after striking it, so a plastic impact can be assumed. Neglecting the energy lost during the impact, determine

(a) the ratio of the maximum belt deflection y_{max} to the static deflection Δ_s with damping neglected, and

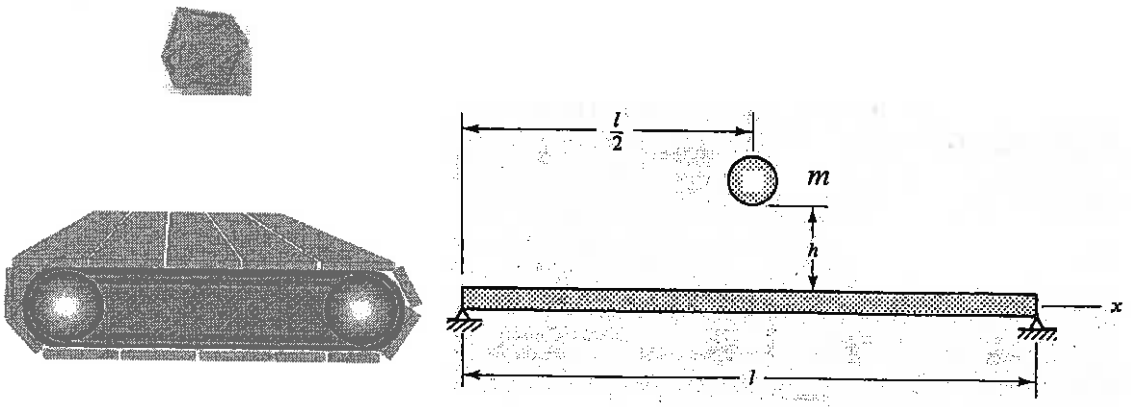
[10 marks]

(b) the same ratio if the damping factor is $\varsigma = 0.1$.

[12 marks]

(c) Comparing the answer in parts (a) and (b), what kind of conclusion can you make?

[3 marks]



(i) (ii)

Figure Q4

Continued .....

Question 5

Figure Q5(i) shows a speed bump in a residential area. It is used to force drivers to slow down. Figure Q5(ii) shows a car, which can be modeled as a spring-mass system, is going over the speed bump. The car is traveling at a constant speed of v. The bump can be modeled as a half sine which has the form of $y(s) = Y\sin(\pi s/\delta)$ where s, δ and Y are the horizontal distance variable, the length and the height of the bump, respectively.

(a) Show that the equations of motion of the car and of the bump in terms of time t can respectively be expressed as

$$m\ddot{z} + kz = -m\ddot{y} \text{ and } y(t) = \begin{cases} Y\sin(\pi t/t_0) & 0 \le t \le t_0 \\ 0 & t > t_0 \end{cases}$$

where t_0 is the time for the car to finish going over the bump and $z = x - y$, x and y are the displacements of the car and the bump, respectively.

[6 marks]

(b) Show that relative response z of the car for the time $0 \le t \le t_0$ is

$$z = \frac{Y\omega_b^2}{\omega_n}\left[\frac{\omega_b}{\left(\omega_b^2 - \omega_n^2\right)}\right]\left\{\sin\omega_n t - \frac{\omega_n}{\omega_b}\sin\omega_b t\right\} \text{ where } \omega_b = \frac{\pi v}{\delta}$$

Hint: use identity $\sin(a)\sin(b) = (1/2)\cos(a-b) - (1/2)\cos(a+b)$.

[13 marks]

(c) Show that the maximum relative deflections of the car when the car is traveling over the bump at high speed $(\omega_b \gg \omega_n)$ and low speed $(\omega_b \ll \omega_n)$ are

$$z_{\max} = \frac{Y\omega_b^3}{\omega_n\left(\omega_b^2 - \omega_n^2\right)} \quad \omega_b \gg \omega_n, \text{ and } z_{\max} = \frac{Y\omega_b^2}{\omega_n^2 - \omega_b^2} \quad \omega_b \ll \omega_n, \text{ respectively.}$$

[6 marks]

Figure Q5(i)

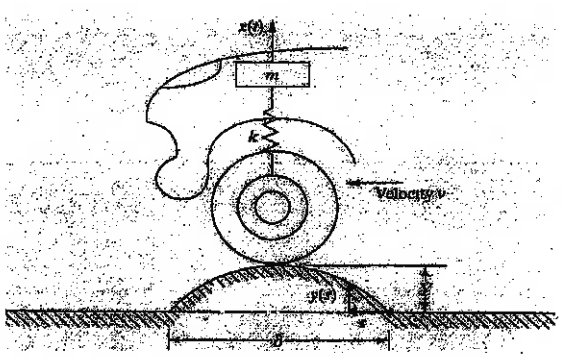


Figure Q5(ii)

Continued .....

Appendix 1

Vibrations

Equations of Motion

$$m\ddot{x} + c\dot{x} + kx = F_0 \sin \omega t$$

General Solution

$$x(t) = x_c(t) + x_p(t)$$

$$x_p(t) = D \sin(\omega t - \varphi)$$

$$D = \frac{F_0 / k}{\sqrt{[1 - r^2]^2 + (2\zeta r)^2}} \qquad where \quad r = \frac{\omega}{\omega_n}$$

$$\varphi = \tan^{-1} \frac{2\zeta r}{1 - r^2}$$

$$x_c(t) = D_1 e^{\lambda_1 t} + D_2 e^{\lambda_2 t} \qquad \zeta > 1$$

$$x_c(t) = (B + Ct) e^{-\omega_n t} \qquad \zeta = 1$$

$$x_c(t) = e^{-\zeta \omega_n t} (B \cos \omega_d t + C \sin \omega_d t) \qquad \zeta < 1$$

$$\omega_n = \sqrt{k/m} = 2\pi f_n, \qquad f_n = \frac{1}{\tau_n} = \frac{\omega_n}{2\pi}$$

$$\zeta = \frac{c}{2m\omega_n}$$

$$\lambda_{1,2} = -\zeta \omega_n \pm \omega_n \sqrt{\zeta^2 - 1} \qquad \zeta \geq 1$$

$$\omega_d = \omega_n \sqrt{1 - \zeta^2} \qquad \zeta < 1$$

Force and Displacement Transmissibility

$$\frac{F_T}{F_0} = \frac{X}{Y} = \left[\frac{1 + (2\zeta r)^2}{(1 - r^2)^2 + (2\zeta r)^2} \right]^{\frac{1}{2}}$$

Rotating Unbalance: Amplitude of Response

$$X = \frac{m_0 e}{m + m_0} \frac{r^2}{\sqrt{(1 - r^2)^2 + (2\zeta r)^2}}$$

Fourier's series

$$x(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} [a_n \cos n\omega t + b_n \sin n\omega t]$$

$$a_n = \frac{2}{\tau} \int_0^{\tau} x(t) \cos n\omega t \, dt \qquad b_n = \frac{2}{\tau} \int_0^{\tau} x(t) \sin n\omega t \, dt$$

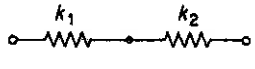
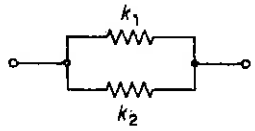
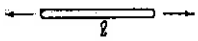
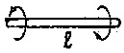
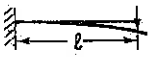
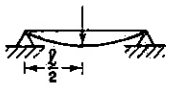
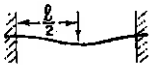
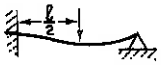
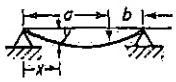
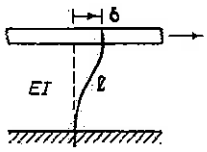
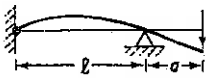
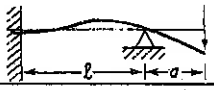
Unit response function (underdamped)

$$h(t - \tau) = \frac{e^{-\xi \omega_n (t - \tau)}}{m\omega_d} \sin \omega_d (t - \tau)$$

Continued .....

Appendix 2

Table of Spring Stiffness

	$k = \dfrac{1}{1/k_1 + 1/k_2}$	
	$k = k_1 + k_2$	
	$k = \dfrac{EI}{l}$	I = moment of inertia of cross-sectional area l = total length
	$k = \dfrac{EA}{l}$	A = cross-sectional area
	$k = \dfrac{GJ}{l}$	J = torsion constant of cross section
	$k = \dfrac{Gd^4}{64nR^3}$	n = number of turns
	$k = \dfrac{3EI}{l^3}$	k at position of load
	$k = \dfrac{48EI}{l^3}$	
	$k = \dfrac{192EI}{l^3}$	
	$k = \dfrac{768EI}{7l^3}$	
	$k = \dfrac{3EIl}{a^2b^2}$	$y_x = \dfrac{Pbx}{6EIl}(l^2 - x^2 - b^2)$
	$k = \dfrac{12EI}{l^3}$	
	$k = \dfrac{3EI}{(l+a)a^2}$	
	$k = \dfrac{24EI}{a^2(3l+8a)}$	

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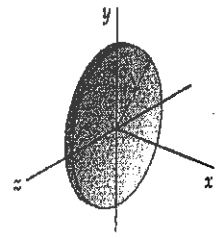
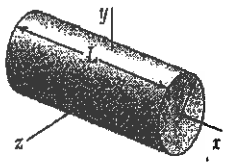
Appendix 3: Mass Moment of Inertia of Homogenous Solids



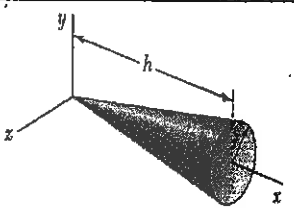
$I_x = \frac{1}{2}mr^2$

$I_y = I_z = \frac{1}{4}mr^2$



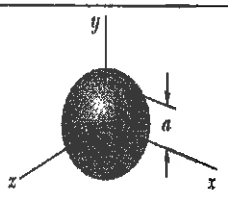
$I_x = \frac{1}{2}ma^2$

$I_y = I_z = \frac{1}{12}m(3a^2 + L^2)$

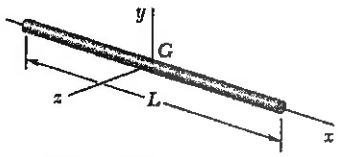


$I_x = \frac{3}{10}ma^2$

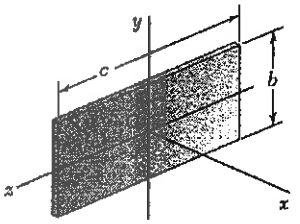
$I_y = I_z = \frac{3}{5}m(\frac{1}{4}a^2 + h^2)$



$I_x = I_y = I_z = \frac{2}{5}ma^2$



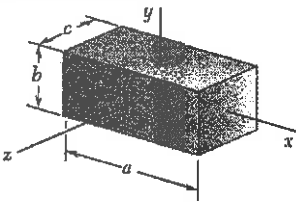
$I_y = I_z = \frac{1}{12}mL^2$



$I_x = \frac{1}{12}m(b^2 + c^2)$

$I_y = \frac{1}{12}mc^2$

$I_z = \frac{1}{12}mb^2$



$I_x = \frac{1}{12}m(b^2 + c^2)$

$I_y = \frac{1}{12}m(c^2 + a^2)$

$I_z = \frac{1}{12}m(a^2 + b^2)$

End of Paper